

**Instructions:** Complete each of the following exercises for practice.

1. Use the Lagrange multipliers method to compute the extreme values of  $f$  subject to the constraint.
  - (a)  $f(x, y) = xy$ ;  $4x^2 + y^2 = 8$
  - (b)  $f(x, y, z) = x^4 + y^4 + z^4$ ;  $x^2 + y^2 + z^2 = 1$
  - (c)  $f(x, y, z) = \ln(x^2 + 1) + \ln(y^2 + 1) + \ln(z^2 + 1)$ ;  $x^2 + y^2 + z^2 = 12$
2. Find the extreme values of  $f$  subject to the constraints.
  - (a)  $f(x, y, z) = x + y + z$ ;  $x^2 + z^2 = 2$ ,  $x + y = 1$
  - (b)  $f(x, y, z) = xy + yz$ ;  $xy = 1$ ,  $y^2 + z^2 = 1$
  - (c)  $f(x, y, z) = x^2 + y^2 + z^2$ ;  $x - y = 1$ ,  $y^2 - z^2 = 1$
3. Find the points on the curve of intersection of the plane  $x + y + 2z = 2$  and the paraboloid  $z = x^2 + y^2$  which are nearest the origin and farthest from the origin.